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LETTER TO THE EDITOR

Classical phase-space structure of the single-particle motion in cranked potentials

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Abstract. The phase-space structure of the single-particle motion in three-dimensional cranked Buck-Pilt potentials of lemniscatoidic shape has been investigated in terms of Poincaré surfaces of section, the mean value of positive maximal Ljapunov exponents, and the chaotic fraction of phase space as a function of the rotational frequency. Due to the rotation of a partial regularization of the particle motion was found for certain values of deformation and frequency.

The investigation of the mixing of regular and chaotic aspects of nuclear motion and the search for its manifestation in various observables involving different degrees of freedom are current subjects of growing interest (for a review see [1]). Based on numerical calculations within the framework of realistic models as well as on comparisons of predicted statistical distributions with available (mostly scarce) experimental data, a general goal of these efforts is to identify the signature of quantum chaos in the dynamics of small quantum systems which behave irregularly in the classical limit.

From measured reaction data on neutron and proton resonances in medium and heavy nuclei one can state that the observed spectral fluctuation properties expressed by the nearest-neighbour level-spacing distribution and the Dyson–Metha Δ_3 statistics are consistent with the predictions of the random matrix theory [2-6]. According to the conjecture [7] that a fully chaotic system with time-reversal symmetry is associated with the Gaussian orthogonal ensemble (GOE) of random matrices this result allows for the conclusion that the dynamics underlying the internal motion of compound nuclei is generally chaotic. But, deduced from the features of highly excited compound nuclei such a statement is first of all restricted to the energy domain near to the particle-decay threshold, a region of high nuclear level density. So the question arises as to what extent chaotic behaviour can prevail also in the lower part of the nuclear spectrum where the nucleonic motion is governed by the mean field, and appropriate concepts to interpret individual states are provided by the shell model and the collective model.

In treating this problem from the experimental side, Abul-Magd and Weidenmüller [8] analysed a rather small sample of data on low-lying levels in nuclei ranging from ^{24}Na to ^{244}Am with the crude indication that only rotational states in even–even nuclei seem to behave rather regularly. From the theoretical side, the onset of chaos in rapidly rotating nuclei has been studied by Aberg [9] on a microscopic basis by adding a

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schematic two-body residual interaction to a cranked Nilsson Hamiltonian. The computed Δ_3 statistics for the energy levels labelled by a certain spin as well as the fragmentation of the rotational $B(E2)$ transition strength indicate that, e.g., for $I = 50^+$ states a smooth transition to chaos proceeds in the excitation energy interval 3-3.5 MeV above the yrast line as the strength of the residual interaction reaches a value which agrees with the average level spacing between neighbouring 2p2h states. Further studies [10–12] of order-to-chaos transitions for collective nuclear states have been performed within the interacting boson model (IBM). In particular, Alhassid *et al* [11, 12] investigated a family of Hamiltonians in which an interpolation between the limits of rotational and γ-unstable even–even nuclei is realized by varying a single parameter. In the limiting cases the system is completely integrable due to the dynamical symmetries contained in the model, but in the transition region the onset of chaos is observed in both the level and the $B(E2)$ statistics. The chaotic regime is more gradually approached from the side of rotational nuclei (SU(3) limit). Performing the semiclassical limit of the IBM, it has been shown that the classical measures of chaos (average maximal Ljapunov exponent, chaotic volume of the phase space) are perfectly correlated with the quantal ones.

The theoretical basis for the treatment of nuclear rotational motion is provided by the cranking model [13]. In the framework of the cranking model various nuclear properties are determined by the response of the nucleons confined in a potential to a rotation with constant frequency ω about an axis perpendicular to the symmetry axis. So, the investigation of the phase-space structure of the three-dimensional single-particle motion in a cranked realistic potential turns out to be a basic task in the discussion of chaos in many-body systems.

The rotation of (two-dimensional) classical billiards with angular velocity perpendicular to the billiard table has been studied for a circular surface rotating about a point on its edge by Fairlie and Siegwart [14] (for the quantal analogue see also [15]) and for an elliptic boundary rotating around its centre by Frisk and Arvieu [16]. Frisk and Arvieu gave also some results for the rotating stadium.

The present letter extends the investigations of Milek and Reif [17] on classical versus quantum chaos in the single-particle motion in strongly deformed fields by analysing the classical phase-space structure of a cranked realistic potential in three dimensions. The finite-depth potential well utilized is the Buck–Pilt ansatz [18] with equipotential surfaces deformed according to lemniscatoidic shapes symmetric with respect to the z-axis

$$V(u) = -V_0 \frac{1+\cosh(R_0/a)}{\cosh(u/a)+\cosh(R_0/a)} \tag{1}$$

$$u = R_0 \frac{x^2+y^2+z^2}{\sqrt{B^2(x^2+y^2)+C^2z^2}}. \tag{2}$$

The deformation parameter $\delta = B/C$ ranges from 1 to 0, covering a family of shapes between a single sphere and two touching spheres, respectively, as it takes place in nuclear fission processes and heavy-ion reactions. Numerical calculations of classical single-particle trajectories have been performed for a heavy nucleus ($A = 252$, $R_0 = 1.2A^{1/3}$ fm, $V_0 = 52$ MeV, $a = 0.66$ fm) with various shapes of the potential given by $\delta = 0.9$ (weakly quadrupole deformed), 0.8, 0.6 (appreciable neck formation). The choice of the binding energy of the particle according to $\eta = (V_0 - |E|)/V_0 = 0.625$ fixes the frequency Ω of the single-particle motion, $\omega = E/\hbar = 19.5$ MeV/$\hbar$. The Newtonian equations of motion in the rotating system have been integrated by a standard Runge–Kutta method for fixed frequencies ω in the range $|\omega| = 0$ up to $|\omega| = 1.6$ of clockwise ($\omega < 0$) as well as counterclockwise ($\omega > 0$) rotation about the x-axis. Using a moment of inertia of $\Im \approx 50\hbar^2\,\mathrm{MeV}^{-1}$ the interval of rotational frequencies implies high-spin states up to $I \approx 80\hbar$. The chosen initial conditions realize a single-particle angular momentum with a fixed z-component ($L_z = 2\hbar$), a y-component equal to zero and an x-component directed in the positive x direction with a magnitude following from the considered binding energy.

Following Arvieu *et al* [19], in accordance with the symmetry of the problem the phase-space portrait in terms of Poincaré surfaces of sections (PSS) has been represented for the intrinsic system in terms of cylindrical coordinates ϱ', $v_{\varrho'} = p_{\varrho'}/m$ (m is the neutron mass) with respect to the equatorial plane ($z' = 0, p_{z'} > 0$) and additionally, in order to complete the visualization of the trajectories, also with respect to a plane containing the symmetry axis ($y' = 0, p_{y'} > 0$). Referring to the topology of the PSS for the static case, $\omega = 0$, $L_z = \text{const}$, for the regular trajectories only restricted parts of the invariant curves associated with crossings near to the central region of the potential are replaced by a stochastic distribution of points if the rotation proceeds with low frequency. This change in the phase-space portrait is created by an additional curvature of the trajectories due to the inertial forces added in the rotating system. Increasing the rotational frequency, chaos is produced gradually for the whole available phase space, the most stable orbits appearing to be the trajectories near to the fixed point of the map (geodesics) (see figure 1). The clockwise rotation with the collective angular momentum and the x-component of the initial single-particle angular momentum oriented in opposite directions turns out to be more effective in generating a transition from regular to chaotic motion than a counterclockwise rotation (compare figures 1(c) and 1(d)).

Obviously, the PSS are insufficient to deduce clear statements on the character of the trajectory or the degree of stochasticity of the phase space, presumably even if one would employ more convenient coordinates as done by Frisk and Arvieu [16] for the billiard. Therefore, the maximal Ljapunov exponent λ has been computed [17] in the rotating system in order to characterize the behaviour of neighbouring trajectories. The evolution of the time limit appearing in the definition of λ has been followed in general up to $t_{\max} \approx 2 \times 10^5\,\mathrm{fm}/c$ and the values of $\lambda(t_{\max}) \leq 5.5 \times 10^{-4}\,c/\mathrm{fm}$ have been interpreted as indicating a regular trajectory ($\lambda = 0$). In figure 2 the order-to-chaos transition due to growing cranking frequencies is expressed by the function $\lambda(t)$ for 10 trajectories which all are regular in the static case. One observes that for a low rotational frequency of $\omega = 0.2\,\mathrm{MeV}/\hbar$ (figure 2(a)) five trajectories remain stable ($\lambda(t_{\max}) \approx 5 \times 10^{-4}\,c/\mathrm{fm}$) while five orbits become irregular ($\lambda(t_{\max}) > 2 \times 10^{-3}\,c/\mathrm{fm}$). For a faster rotation, $\omega = 0.8\,\mathrm{MeV}/\hbar$ (figure 2(b)), all trajectories are chaotic. But it should be mentioned that in other cases also trajectories exist the large positive Ljapunov exponent of which is lowered by a rotation of the potential with certain frequency.

In order to characterize the phase-space organization as a function of the cranking frequency in a more quantitative manner the maximal Ljapunov exponents have been calculated for a grid of initial conditions in the available phase space including about 350 mesh points. Then, one can deduce an average positive Ljapunov exponent $\bar{\lambda}$ and a chaotic volume μ according to $\mu = N_c/(N_c + N_r)$, where N_c and N_r denote the number of initial conditions leading to chaotic and regular trajectories, respectively. Due to the finite time limit in the calculations a few values of $\lambda(t_{\max})$ come out close to the

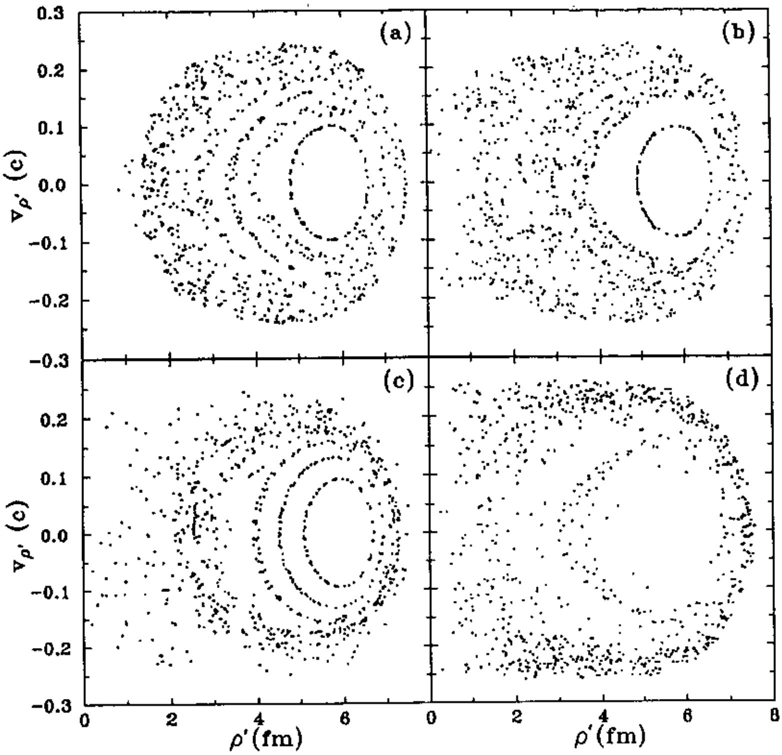

Figure 1. Poincaré surfaces of section ($z'=0, p_{z'}>0$) generated by 10 trajectories with initial angular momentum components $L_x>0$, $L_y=0$, $L_z=2\hbar$ for deformation parameter $\delta=0.9$ and various rotational frequencies ω. (a): $\omega=0.2$ MeV/$\hbar$, (b): $\omega=0.4$ MeV/$\hbar$, (c): $\omega=0.8$ MeV/$\hbar$, (d): $\omega=-0.8$ MeV/$\hbar$. For the static case ($\omega=0$) the phase space is regular.

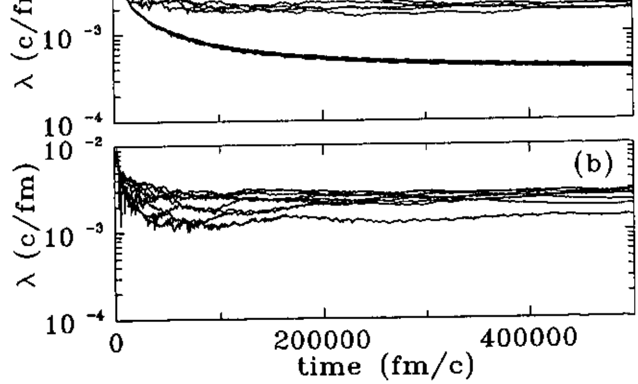

Figure 2. Time dependence of the numerically calculated Ljapunov exponents λ for 10 trajectories behaving regular in the static case ($\omega=0$). Initial angular momentum: $L_x>0$, $L_y=0$, $L_z=2\hbar$. (a): $\omega=0.2$ MeV/$\hbar$, (b): $\omega=0.8$ MeV/$\hbar$.

number below which a vanishing Ljapunov exponent is expected for $t \to \infty$. This situation gives rise to some ambiguities in counting the number of regular and chaotic initial conditions. In order to allow for this circumstance, $\bar{\lambda}$ and μ have been computed taking into account only values $\lambda(t_{max})$ larger than some appropriately chosen values $\lambda_{min}(t_{max})$ in determining N_c. For $\lambda_{min}(t_{max}) = 7 \times 10^{-4}\, c/\mathrm{fm}$ the chaotic volume μ as a function of ω is shown in figure 3(*a*) for various deformation parameters.

One observes that starting out from the value for the static case the chaotic fraction of the phase space increases with growing $|\omega|$, but for rapid rotations reaches a saturation value, which rises with the shape deformation. For necked-in potentials ($\delta = 0.6$) a saturation value of $\mu \approx 1$ is achieved already for a very low rotational frequency. For weakly deformed potentials an oscillatory structure is superimposed, the relative minima of which indicate a relative stabilization of regular orbits for certain ratios of the single-particle and the collective frequency. This behaviour is different for clockwise and counterclockwise rotation of the potential. Furthermore, for quadrupole deformed fields the clockwise rotation (L_x-component of the initial single-particle and collective angular momentum directed antiparallel) leads to a higher saturation value for the chaotic value compared with the counterclockwise rotation.

From detailed analysis one finds out that a chaos-to-order transition due to growing cranking frequency happens for trajectories starting near to the fixed point or for

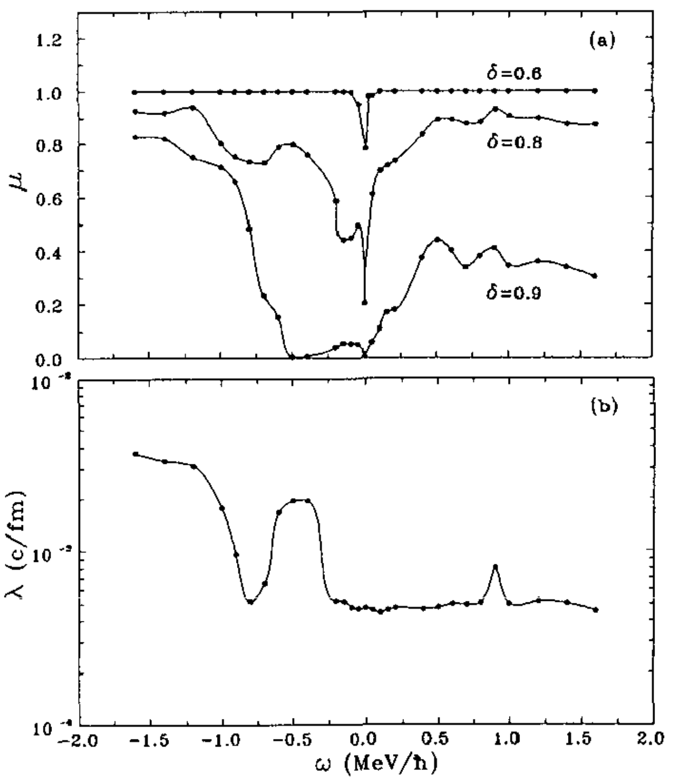

Figure 3. (*a*) Chaotic volume μ as a function of the cranking frequency ω for various deformation parameters δ. Initial orbital angular momentum: $L_x > 0$, $L_y = 0$, $L_z = 2\hbar$. (*b*) Maximal positive Ljapunov exponent λ in dependence of ω. Deformation parameter: $\delta = 0.8$, initial condition: $\varrho = 5.7$ fm, $v_\varrho = 0.012c$, initial angular momentum: $L_x > 0$, $L_y = 0$, $L_z = 2\hbar$.

bouncing-ball trajectories. This behaviour is demonstrated in figure 3(*b*) which shows the maximal Ljapunov exponent λ as a function of ω for an initial condition selected close to the fixed point. But, it should be stressed that for the present potential with its smooth surface region the stabilization of regularity is less pronounced than it was found [16] for the elliptic billiard or the stadium.

The ω dependence of the average Ljapunov exponent $\bar{\lambda}$ shows the same properties.

The present investigations of the classical single-particle motion in a cranked potential has to be carried on by the analysis of the spectral properties of the quantum analogue. For the nearest-neighbour level-spacing distribution one expects a level repulsion according to the Gaussian unitary ensemble because of the broken time reversal symmetry of the problem. Further, it should be necessary to work out the consequences of the classical phase-space organization to the dynamical properties of nuclei as moments of inertia, transition rates, shape and angular momentum dependence of viscosity, or the centrifugal solidifaction of superdeformed nuclei [20]. For a rotating elliptic billiard Traiber *et al* [21] found out already that irregularities in the behaviour of the dynamical moment of inertia of a quantum many-particle system are associated with the chaotic dynamics of the corresponding classical system. The influence of the coexistence of regular and chaotic aspects of nuclear dynamics on the time evolution of nuclei in heavy-ion reactions and fission processes could be of particular interest [22].

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